GOSFORD HIGH SCHOOL
Mathematics Assessment Task No 1
December 2006

- **Time allowed - 60 minutes plus 5 minutes reading time**
- **All necessary working must be shown**
- **Full marks may not be awarded for untidy work or work that is poorly set out**
- **Begin each new question on a new page**

Question 1

a) The quadratic equation $x^2 - 3x - 5 = 0$ has roots α and β. What is the value of:

i) $\alpha + \beta$ **(1)**

ii) $\alpha\beta$ **(1)**

iii) $\alpha^2 + \beta^2$ **(2)**

b) A quadratic equation has roots α and β. Write down the equation if $\alpha + \beta = 3$ and $\alpha\beta = -1$. **(2)**

c) Find the value of k in $x^2 + 6x + k = 0$ if one root is double the other. **(2)**

d) Find the values of k for which $x^2 + 6x - 3k = 0$ has real roots. **(2)**

e) Find the minimum value of $2x^2 - 12x + 7$. **(2)**

Question 2

a) Solve $3x^2 - 5x + 1 = 0$ expressing answers in simplest surd form. **(2)**

b) Solve $x^4 - 10x^2 + 9 = 0$ **(3)**

c) Differentiate

i) $x\sqrt{x}$ **(1)**

ii) $(3x^2 - 7)^{10}$ **(1)**

iii) $\dfrac{2x-5}{3x+7}$ **(2)**

iv) $2x(3-x)^4$ **(2)**

Question 3

a) Find exact values for p if $px^2 - 2x + 3p = 0$ is negative definite. (3)

b) Evaluate $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3}$ (2)

c) The curve $y = x^3 + ax^2 + 7x - 5$ has a stationary point when $x = 1$.
Find the value of a. (2)

d) Find $\dfrac{d^2y}{dx^2}$ if $y = 3x^4 - x^3$ (2)

e) A continuous curve $y = f(x)$ has the following properties for the interval $a \le x \le b$:

$$f(x) > 0, \quad f'(x) > 0 \text{ and } f''(x) < 0$$

Sketch a curve satisfying these conditions and state the greatest value of $f(x)$ in this interval. (3)

Question 4

a) i) Show that the gradient of the tangent to the curve $y = \dfrac{1}{4}x^2(x^2 - 7)$ at the point A $(2,-3)$ is 1. Hence find the equation of this tangent. (4)

ii) Determine the equation of the tangent at the point B $(-2,-3)$ on the curve. (3)

iii) Show that the angle between the tangents at A and B is $90°$ and calculate the point of intersection of these tangents. (2)

b) Find the equation of the normal to the parabola $y = 3 + 6x - 2x^2$ at the point where $x = 1$. (3)

Question 5

a) Given $f(x) = x^3 - 6x^2 + 9x - 5$, find for what values of x the function is:

i) Increasing (2)

ii) concave up (2)

b) Find any stationary points on this curve and determine their nature. (4)

c) Neatly sketch the curve for $0 \le x \le 5$, clearly labelling all critical points and the y intercept. (3)

d) What are the greatest and least values of the function in the interval $0 \le x \le 5$? (2)

Question 6

a) For the parabola $y=\frac{1}{6}x^2$, find:

i) the co-ordinates of the focus **(2)**

ii) the equation of the directrix **(1)**

iii) the length of the latus rectum **(2)**

b) i) A and B are the points $(-3,-1)$ and $(7,3)$ respectively.
The point P(x, y) moves so that $\angle APB=90°$.
Show that the equation of the locus of P is the circle with equation
$x^2-4x+y^2-2y=24$. **(2)**

ii) State the co-ordinates of the centre and the length of the radius of this circle. **(2)**

c) Find the co-ordinates of the vertex and the focus of the parabola $2y=(x-1)^2-4$ **(2)**

2006 - 2U Task 1

1 a) $x^2 - 3x - 5 = 0$

i) $\alpha + \beta = \frac{-(-3)}{1} = 3$

ii) $\alpha\beta = -5$

iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (3)^2 - 2(-5)$

$= 9 + 10$

$= 19$

b) $\alpha + \beta = 3$, $\alpha\beta = -1$

$x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$\therefore\ x^2 - 3x - 1 = 0$

c) $x^2 + 6x + k = 0$

let roots be $\alpha, 2\alpha$

$\therefore\ \alpha + 2\alpha = -6$

$3\alpha = -6$

$\alpha = -3$

$2\alpha^2 = k$

$\therefore\ k = 2(-3)^2$

$= 18$

d) $x^2 + 6x - 3k = 0$

real roots $\Rightarrow \Delta \geq 0$

$\Delta = 36 + 12k$

$\therefore\ 36 + 12k \geq 0$

$12k \geq -36$

$k \geq -3$

e) $2x^2 - 12x + 7$

vertex at $x = \frac{-b}{2a}$

$= \frac{12}{4}$

$= 3$

$\therefore$ min. value at $2(9) - 36 + 7$

$= -11$

Q2

a) $3x^2 - 5x + 1 = 0$

$x = \frac{5 \pm \sqrt{25-12}}{6}$

$= \frac{5 \pm \sqrt{13}}{6}$

b) $x^4 - 10x^2 + 9 = 0$

let $m = x^2$

solve $m^2 - 10m + 9 = 0$

$(m-9)(m-1) = 0$

$m = 9$, $x^2 = 9$, $x = \pm 3$

$m = 1$, $x^2 = 1$, $x = \pm 1$

c) i) $\frac{d}{dx}\left(x^{3/2}\right) = \frac{3}{2}x^{1/2}$

$= \frac{3\sqrt{x}}{2}$

ii) $\frac{d}{dx}(3x^2-7)^{10} = 10(3x^2-7)^9(6x)$

$= 60x(3x^2-7)^9$

iii) $\frac{d}{dx}\left(\frac{2x-5}{3x+7}\right)$

$= \frac{(3x+7)(2) - (2x-5)(3)}{(3x+7)^2}$

$= \frac{6x + 14 - 6x + 15}{(3x+7)^2}$

$= \frac{29}{(3x+7)^2}$

iv) $\frac{d}{dx}\left(2x(3-x)^4\right)$

$u = 2x \qquad v = (3-x)^4$

$u' = 2 \qquad v' = 4(3-x)^3(-1)$

$\qquad\qquad = -4(3-x)^3$

$= (3-x)^4(2) - (2x)4(3-x)^3$

$= 2(3-x)^3\left[(3-x) - 4x\right]$

$= 2(3-x)^3(3-5x)$

<u>Q3</u>

a) $px^2 - 2x + 3p = 0$

neg. def $\quad p<0 \quad , \Delta < 0$

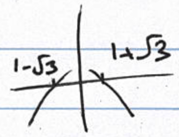
$\Delta = 4 - 4(p)(3p)$

$= 4 - 12p^2$

$\therefore \; 4 - 12p^2 < 0$

$4(1-3p^2) < 0$

$4(1-\sqrt{3}p)(1+\sqrt{3}p) < 0$



$\therefore \; p < 1-\sqrt{3}$

b) $\lim_{x\to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}}$

$= 3+3$

$= 6$

c) $y = x^3 + ax^2 + 7x - 5$

$y' = 0$ at $x = 1$

$y' = 3x^2 + 2ax + 7$

$\therefore \; 0 = 3 + 2a + 7$

$\therefore \; 2a = -10$

$a = -5$

d) $y = 3x^4 - x^3$

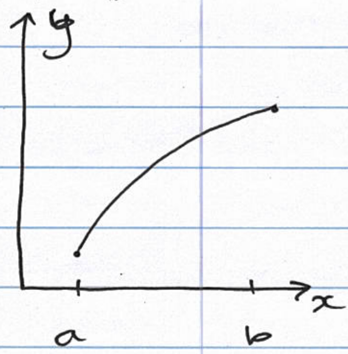
$y' = 12x^3 - 3x^2$

$\frac{d^2y}{dx^2} = 36x^2 - 6x$

e) $f(x) > 0 \rightarrow$ above x axis

$f'(x) > 0 \rightarrow$ +ve gradient $\rightarrow$ rising

$f''(x) < 0 \rightarrow$ concave down



greatest value $f(b)$

<u>Q4</u>

a) i) $y = \frac{1}{4}x^2(x^2-7)$

$y = \frac{1}{4}x^4 - \frac{7}{4}x^2$

$y' = x^3 - \frac{7}{2}x$

at $x = 2$

m of tangent $= (2)^3 - (\frac{7}{2})(2)$

$= 8 - \frac{7}{2} \times 2$

$= 8 - 7$

$= 1$

$\therefore$ eqn of tangent

$y + 3 = 1(x-2)$

$y + 3 = x - 2$

$y = x - 5$

ii) at $x = -2$

m of tangent $= (-2)^3 - \frac{7}{2}(-2)$

$= -8 + 7$

$= -1$

$\therefore$ eqn of tangent

$y+3=-1(x+2)$

$y+3=-x-2$

$x+y+5=0$

iii) m_1 of tangent at $A=1$

m_2 of tangent at $B=-1$

$\therefore m_1 \times m_2 = 1 \times -1$

$= -1$

$\therefore$ tangent at A $\perp$ tangent at B

& $\therefore$ at $90°$ to each other.

pt. intersection at:

$x+(x-5)+5=0$

$2x=0$

$x=0$ $\quad (0,-5)$

$\therefore y=-5$

b) $y=3+6x-2x^2$ at $x=1$ normal

$y'=6-4x$

at $x=1$ m of tangent $=6-4$

$=2$

$\therefore$ m of normal $=-\frac{1}{2}$

$x=1,\ y=3+6-2$

$y=7$ $\quad (1,7)$

$\therefore$ eqn normal

$y-7=-\frac{1}{2}(x-1)$

$2y-14=-x+1$

$x+2y-15=0$

Q5

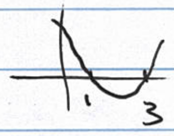
a) $f(x)=x^3-6x^2+9x-5$

i) increasing $f'(x)>0$

$f'(x)=3x^2-12x+9$

$=3(x^2-4x+3)$

$=3(x-3)(x-1)$



$\therefore$ increasing at $x>3,\ x<1$

ii) concave up $f''(x)>0$

$f''(x)=6x-12$

$\therefore 6x-12>0$

$6x>12$

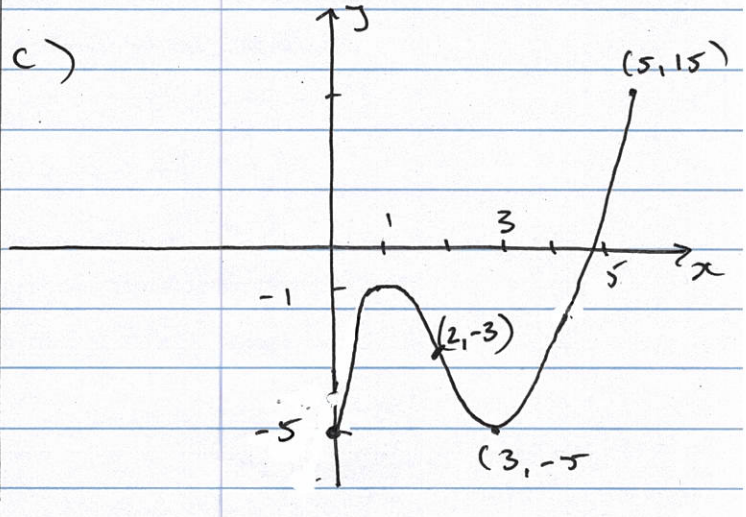
$x>2$

b) stat pts at $f'(x)=0$.

$3(x-3)(x-1)=0$

$x=3$	$x=1$
$y=-5$	$y=-1$
$f''(3)=6>0$	$f''(1)=-6<0$
$\therefore$ minimum turning pt	$\therefore$ maximum turning pt
at $x=3$	at $x=1$, $y=-1$
$y=-5$	

c)



d) greatest value $= 15$

least value $= -5$

Q6

a) $y = \frac{1}{6}x^2$

i) $4a = \frac{1}{6}$

$a = \frac{1}{24}$

$\therefore$ focus $(0, \frac{1}{24})$

ii) directrix $y = -\frac{1}{24}$

iii) length of latus rectum $= 4 \times \frac{1}{24}$

$= \frac{1}{6}$

b) i) $A(-3,-1)$, $B(7,3)$ $P(x,y)$

$m(AP) \perp m(BP)$

$m(AP) = \frac{y+1}{x+3}$

$m(BP) = \frac{y-3}{x-7}$

d $\frac{y+1}{x+3} \times \frac{y-3}{x-7} = -1$

$\therefore y^2 - 3y + y - 3 = -(x^2 - 7x + 3x - 21)$

$y^2 - 2y - 3 = -x^2 + 4x + 21$

$(x^2 - 4x + 4) + (y^2 - 2y + 1) = 21 + 3 + 4 + 1$

$(x-2)^2 + (y-1)^2 = 29$

or $x^2 - 4x + y^2 - 2y = 24$

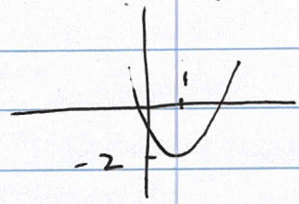
ii) centre $(2,1)$ radius $= \sqrt{29}$

c) $2y = (x-1)^2 - 4$

$2y + 4 = (x-1)^2$

$2(y+2) = (x-1)^2$

$\therefore$ vertex $(1,-2)$ $a = \frac{1}{2}$



focus $(1, -1\frac{1}{2})$